Review of the book

Discrete Chaos, second edition

by Saber N. Elaydi

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1 Overview of Book

This is the second edition of an introductory text in discrete dynamical systems written by a successful researcher and expositor in dynamical systems/difference equations. This new edition maintains most of the topics presented in the first edition, but also includes more recent results on global stability, bifurcation, chaos and fractals, as well as newer applications to biology, chemistry and physics. A CD of an adapted version of PHASER is also provided in this new edition, allow students to solve problems given in the text. Suitable for advanced undergraduates and beginning graduate students in mathematics, this text offers a good introduction to the fundamentals of discrete dynamical systems.

2 Book Summary

Chapter 1 contains an exposition of the dynamics of one-dimensional maps. It defines and then provides the criteria for the stability of hyperbolic and non-hyperbolic fixed and periodic points. Towards the end of the chapter, the period-doubling route to chaos is presented. In particular, the logistic map is given in detail, including its fixed points, 2 and 4-periodic cycles. The chapter ends with a presentation of 2 fish population models as an application of the above concepts.

Chapter 2 gives a presentation of the basin of attraction of fixed points and periodic orbits. It then proves Singer's theorem on periodic attractors. The rudiments of bifurcation theory are then presented, with the aid of some examples. The chapter continues with a presentation of Sharkovsky's theorem and includes its proof in an appendix. The author's own work on the converse of this theorem is also given. The chapter ends with a brief mention of Poincare's sections, and one of their applications to chemistry.

Chapter 3 gives a brief introduction to metric spaces, followed by a rigorous presentation of chaos theory. This is facilitated by the introduction of concepts including the density of a set, transitivity and sensitive dependence of a map on an interval, symbolic dynamics and conjugacy. These concepts find applications in Rossler's attractor and in a map which produces a ring pattern qualitatively resembling Saturn's rings.

Chapter 4 gives a thorough presentation of the stability of 2-dimensional maps. This includes an investigation of linear maps and second-order difference equations. Using a method of Liapunov and by linearisation, the stability of nonlinear maps is studied. The chapter then gives a brief presentation to the Hartman-Grobman theorem and the stable manifold theorem. The chapter ends with applications of the above concepts to the kicked rotator and the Henon map, a discrete epidemic model for gonorrhea, and perennial grass.

Chapter 5 starts with a presentation on center manifolds and bifurcation It then presents the chaos of two-dimensional maps. This includes the study of hyperbolic Anosov toral automorphisms, with a presentation of the topological transitivity of the toral automorphism given in the appendix of the chapter. This also includes the study of subshifts of finite type, and the horseshoe of Smale. The Chapter concludes with a interesting case study, using the concepts studied here, on the extinction and sustainability in ancient civilizations.

Chapter 6 gives rigorous and extensive presentation on fractals, generated by affine transformations, and the underlying theory of iterated function system. As an application of fractals, the chapter concludes with a brief presentation on the collage theorem and image compression.

The final chapter studies the dynamics of one-dimensional maps in the complex plane, and may be considered as an extension of Chapter 1 to the complex numbers. The Julia set and its topological properties and then presented, followed by Newton's method in the complex plane and the associated basin of attraction. The chapter concludes by studying the Mandelbrot set and its connection with the bifurcation diagram of the real-valued quadratic map.

3 Reviewer's Comments

This book offers a good self-contained, clear and readable coverage of discrete dynamical systems and difference equations. The author has a gift for making the more difficult concepts accessible to students with varying backgrounds and interests The reader following the book would have attained a comprehensive view of the basics of these topics. This book can be considered as an extension of the author's book, *An Introduction to Difference Equations*, which presents the classical aspects of difference equations.

While most material in this book are concise and come with detailed rigorous proofs, the reader may want to supplement his reading by browsing introductory texts on real analysis, general topology and linear algebra, to become more familiar with these topics used in this text. Doing the exercises also reinforces the reader's understanding of the material covered. A plus point about this text is that solutions to some exercises are sketched at the back of the book, so that the interested reader can be helped along quickly in better understanding concepts in the text.

The inclusion of many applications of the material to biology, chemistry and physics throughout the text is also a plus point. It breathes more life into the otherwise abstract, but still beautiful, mathematics.

4 Reviewer’s Recommendation

I recommend this book to advanced undergraduates and beginning graduate students interested in discrete dynamical systems and difference equations and wishing to read more advanced texts on this topic.

The reviewer is a senior engineer at the Centre for Strategic Infocomm Technologies, Singapore.